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ACOUSTIC WAVE SCATTERING FROM A PROGRESSIVE INTERNAL WAVEFIELD, (U)
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ACOUSTIC WAVE SCATTERING FROM A PROGRESSIVE
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ACOUSTIC WAVE SCATTERING FROM A PROGRESSIVE INTERNAL WAVE FIELD

P. A. Barakos

ABSTRACT

↙ The scattering of sound from an internal wave field is considered. The acoustic index of refraction depends on the density of sea water. Hence, spatial and temporal density variations arising from the presence of internal waves will produce a perturbation in the index of refraction. Therefore, an acoustic wave transmitted through a stratified fluid in which internal waves are present, will be modulated by the internal wave field and scattering and refraction will occur. It is shown that the frequency spectrum of the scattered acoustic field is comprised of the fundamental acoustic frequency and two side frequencies. Equations describing the phenomenon are given. ↘

INTRODUCTION

Propagation of sound in surface and subsurface ducts has received extensive treatment in the literature (1, 2, 3, 4, 5, 6). Recently, Schulkin (7) has summarized our knowledge of this important branch of Ocean Acoustics. Perfect sound channels are strictly horizontal and their effects upon acoustic wave propagation simply reduce to certain anomalies in wave refraction. In the real ocean, however, imperfect sound channels with space - and time - varying boundaries are nearly always a universal phenomenon. The imperfections are caused by sea roughness, bubble layers, variations in density gradients, absorption, turbulence, water currents, current gradients, temperature microstructure, biological effects and other disturbances in the thermocline mostly due to internal waves and turbulence. To elucidate and overcome these imperfections in sound propagation is one of the principal problems of ocean acoustics.

Moreover, the literature on the theory of scattering of acoustic radiation from rough surfaces is very extensive (8, 9) and the problem of acoustic wave propagation in imperfect channels has not been completely solved. Clay (10), Bucker (11) and Bucker and Morris (12) have considered some of the effects of surface roughness on the normal-mode description of duct propagation and find, in general, that sea surface roughness contributes to the attenuation of the normal modes.

The related problem of amplitude and phase modulation of an acoustic wave field from a periodic free surface has been investigated experimentally by Roderick (13) and Roderick and Cron (14) in a model

tank. They show that the frequency spectrum of the forward scattered modulated signal consists of a discrete frequency spectrum that is centered about the carrier, and a continuous spectrum that is positioned symmetrically about the carrier.

Similarly, for back scattering from a free surface in motion, Marsh (15) employing linear theory shows that the reverberation spectra are narrow and centered at frequencies $\omega_0 \pm \Omega$ with $\Omega = (2 \omega_0 g \cos \theta_0 / c)^{\frac{1}{2}}$ in which g is the acceleration due to gravity, θ_0 is the backscattering angle of the reradiation, ω_0 is the angular frequency of the carrier and c is its phase velocity. Lieberman (16) confirms this prediction for sound in air at a carrier frequency of 78 kHz over a randomly agitated water surface. For capillary waves having wavelengths of 0.19 to 0.36 cm (for capillary waves $v = (2\pi \gamma / \Lambda \rho)^{\frac{1}{2}}$, where v is the capillary phase velocity, and Λ is their wavelength, ρ is the density and γ is the surface tension of the water) he predicts a Doppler shift of 167 Hz with a dispersion of 30 Hz about the shift. While Mellen (17) employing carrier frequencies of 85 kHz and 1.4 MHz underwater at the Thames River estuary with wind roughened surfaces failed to observe the predicted Doppler spectrum of surface backscatter. He hypothesizes that the apparent failure of first order linear theory to predict the Doppler spectrum of surface backscatter suggests that nonlinear surface effects may be dominant at these wavelengths. However, one should note that the first order linear theory successfully predicts the Doppler spectrum for both back- and forward scattering for results obtained in model tank

experiments and more recently for results obtained in the ocean, Roderick and Cron (14), at 750 Kz and 1.5 kHz. Nevertheless, it is possible that nonlinear effects may be important.

Herein, consideration will be principally confined to the lower boundary of the imperfect duct along which internal gravity waves may be propagated. The internal wave field will modulate the amplitude of the acoustic wave field traversing the perturbed interface so that the spectrum of the scattered acoustic field contains, along with the fundamental acoustic frequency, two additional side frequencies.

The significance of the present investigation is that the amplitude and phase modulation of the acoustic wave field by internal waves will result in sidebands similar to those observed by Roderick for forward scattering and Lieberman for backscattering but lower in frequency. Consequently in a medium in which both free surface gravity waves and internal waves are present, the resulting spectrum of the scattered frequency will be comprised of the fundamental acoustic frequency and four additional side frequencies; the outer two resulting from surface gravity waves and the inner two from internal waves. In the results of project MIMI, Steinberg (18), there seems to be some experimental evidence, Williams and De-ferrari (19), to support this hypothesis.

To obtain an order of magnitude of the Doppler shift for internal waves, we consider the two layer model. For this model, internal waves have their maximum amplitude at the interface. If the wavelength (λ) is small with respect to the thickness (H) of the upper layer, the phase

velocity of the wave is given by $c = (g\Lambda\Delta\rho/2\pi\rho)^{\frac{1}{2}}$ where g is the acceleration due to gravity, $\Delta\rho$ is the density difference between the juxtaposed layers and ρ is the mean density. On the other hand if the thickness of the upper layer is small with respect to the wavelength, the phase velocity is given by $c = (gH\Delta\rho/\rho)^{\frac{1}{2}}$. In the open ocean $\Delta\rho = 0.002 \text{ gm/cm}^3$ is an upper limit while for other special situations values as high as 0.1 gm/cm^3 , Degens and Ross (20), may be encountered. The distribution of wavelengths (Λ) for internal waves varies widely from a few tens of meters in coastal waters and semi-enclosed basins to a few hundreds of meters to many kilometers for the continental shelves and the open oceans. From these equations, however, one obtains an approximate value of 150 cm/sec for the phase velocity for internal waves. Hence, for acoustic radiation with carrier frequencies in the audible part of the spectrum, Doppler shifts of a few tens of Hertz, at most, are expected.

MATHEMATICAL FORMULATION OF THE PROBLEM

The procedure followed herein is exactly identical to that employed in the diffraction of electromagnetic waves by ultrasonic waves, Morse and Ingard (21) and Born and Wolf (22).

To investigate the scattering of an acoustic wave by an internal wave field, we start with the wave equation for the acoustic field

$$\Delta^2\psi - \frac{1}{c^2} \frac{\partial^2\psi}{\partial t^2} = 0, \quad (1)$$

where Δ is the two-dimensional Laplacian, ψ is the space- and time-dependent acoustic velocity potential and c is the speed of sound in the ocean.

We write for the acoustic index of refraction $n = c_0/c$, where c is the speed of sound at some reference depth of the water and c is the speed of sound at some other depth. We express the index of refraction as the sum of an unperturbed value n_0 and a perturbed value n_1 caused by the internal waves. Since $n_1/n_0 \ll 1$, then $n^2 = n_0^2 + 2n_0n_1$ and the wave equation is expressed approximately as

$$\Delta^2 \psi - \frac{2n_1/n_0}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0. \quad (2)$$

If we introduce the relative variation in the temperature which is related to the index of refraction approximately as $n \sim 1/T$ so that $2\delta n/n = 2T/\delta T$ then Eqn. (2) becomes

$$\Delta^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{(2T/\delta T)}{c^2} \frac{\partial^2 \psi}{\partial t^2}. \quad (3)$$

We assume the coordinate geometry of the internal and acoustic wave fields to be as shown in Figure 1. The internal wave field is assumed to exist along a transition layer having a thickness $2L$.

The internal wave field will be of the form $\zeta(x,t) = b \cos(k_s x - \omega_s t)$ where k_s is the horizontal internal wave number and $\omega_s/2\pi$ is its angular frequency. The perturbation in the index of refraction is of the same form, so we shall write for the temperature field

$$T/\delta T = \alpha \cos(k_s x - \omega_s t) \quad (4)$$

and for the incident acoustic wave

$$\psi(x,z,t) = \psi_0 \cos(\hat{K} \cdot \hat{r} - \Omega t), \quad (5)$$

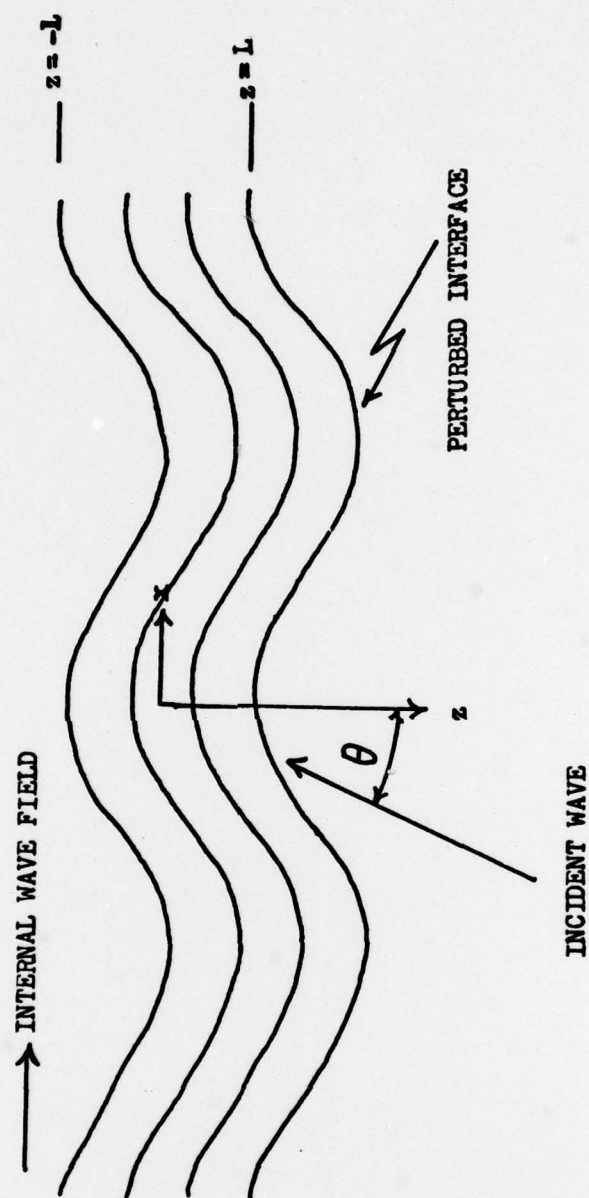


Fig. 1 - Schematic Diagram of the Coordinate System Used in the Discussion of Doppler Shift by Internal Waves

where $\hat{K} \cdot \hat{r} = Kx \sin \theta - Kz \cos \theta$ and ψ_0 is the amplitude of the incident acoustic velocity potential and $f = \Omega/2\pi$ is the frequency of sound.

We take a Fourier transform in time of Eqn. (3) and use

$$F\{f(x)g(x)\} = \int_{-\infty}^{\infty} F(\eta)G(\alpha-\eta)d\eta \quad \text{to obtain}$$

$$\Delta^2 \psi(x, z, \omega) + k^2 \psi(x, z, \omega) = c^{-2} \int_{-\infty}^{\infty} (\omega')^2 \psi(x, z, \omega') f(\omega - \omega') d\omega', \quad (6)$$

where $f(\omega - \omega')$ is the Fourier transform of $T/\delta T$. Similarly, taking Fourier transforms of (4) and (5) we have, respectively,

$$f(\omega - \omega') = \frac{\alpha}{2} \left\{ e^{ikx} \delta(\omega - \omega' - \omega_s) + e^{-ikx} \delta(\omega + \omega' - \omega_s) \right\} \quad (7)$$

and

$$\psi(x, z, \omega) = \frac{\psi_0}{2} \left\{ \delta(\omega' - \Omega) e^{i\hat{K} \cdot \hat{r}} + \delta(\omega' + \Omega) e^{-i\hat{K} \cdot \hat{r}} \right\}. \quad (8)$$

We will employ the Born approximation in assuming that the scattered field is not too different from the incident field. Hence, we substitute Eqns. (7) and (8) for $f(\omega - \omega')$ and $\psi(x, z, \omega')$ in the integrand of Eqn. (6), to obtain

$$\begin{aligned} \Delta^2 \psi + k^2 \psi = & -\frac{\Omega^2 \alpha \psi_0}{4c^2} \left\{ \delta(\omega - \Omega - \omega_s) e^{i(\hat{K} \cdot \hat{r} + kx)} + \delta(\omega - \Omega + \omega_s) e^{i(\hat{K} \cdot \hat{r} - kx)} + \right. \\ & \left. + \delta(\omega + \Omega - \omega_s) e^{-i(\hat{K} \cdot \hat{r} - kx)} + \delta(\omega + \Omega + \omega_s) e^{-i(\hat{K} \cdot \hat{r} + kx)} \right\}. \end{aligned} \quad (9)$$

Eqn. (9) shows that the internal wave field modulates the incident acoustic radiation such that acoustic components with frequencies $\Omega \pm \omega_s$ are generated in the scattering region.

The associated scattered field can be calculated using the two-dimensional Green's function

$$G(x, z | x_0, z_0) = \frac{i}{4} H_0^{(1)}(k \sqrt{(x-x_0)^2 + (z-z_0)^2}) \quad (10)$$

which is a solution to the equation

$$\Delta^2 G + k^2 G = -\delta(\hat{r} - \hat{r}_0) \quad (11)$$

Eqn. (10) represents the field produced by a line source of unit strength at (x_0, z_0) . The scattered acoustic field from the internal wave is then represented as

$$\psi(x, z, \omega) = \int_{-\infty}^{\infty} dx_0 \int_{-L}^L S(x_0, z_0; \omega) G(x, z | x_0, z_0) dz_0, \quad (12)$$

where we define the source $S(x_0, z_0; \omega)$ by the right hand of Eqn. (9), viz.,

$$S(x_0, z_0; \omega) = -\frac{\Omega^2 \alpha \psi_0}{4c^2} \left\{ \delta(\omega - \Omega - \omega_s) e^{i(\hat{K} \cdot \hat{r}_0 + kx_0)} + \right.$$

$$\begin{aligned}
& + \delta(\omega - \Omega + \omega_s) e^{i(\hat{K} \cdot \hat{r}_0 - kx_0)} + \delta(\omega + \Omega - \omega_s) e^{-i(\hat{K} \cdot \hat{r}_0 - kx_0)} + \\
& + \delta(\omega + \Omega - \omega_s) e^{-i(\hat{K} \cdot \hat{r}_0 + kx_0)} \Big\}.
\end{aligned} \tag{13}$$

We employ the Fourier integral transform, Morse and Feshbach (22)

$$\frac{i}{4} H_0^{(1)}(kr) = \frac{1}{4\pi^2} \int dK_x \int dK_z \frac{e^{i[K_x(x-x_0) + K_z(z-z_0)]}}{K_x^2 + K_z^2 - k^2} \tag{14}$$

to replace the Hankel function in Eqn. (12). The K_z path of integration is chosen so as to lead to an outgoing wave from the source point $r = 0$.

Integrating Eqn. (14) we obtain

$$\frac{i}{4} H_0^{(1)}(kr) = \frac{i}{4\pi} \int dK_x \frac{e^{i[K_x(x-x_0) + \sqrt{k^2 - K_x^2}(z-z_0)]}}{\sqrt{k^2 - K_x^2}} \tag{15}$$

and the inverse Fourier transform of this is found to be

$$\frac{i}{8\pi} \int_{-\infty}^{\infty} e^{i\mu x_0} H_0^{(1)}(kr) dx_0 = \frac{i}{4\pi} \frac{e^{i[\mu x + \sqrt{k^2 - \mu^2}(z-z_0)]}}{\sqrt{k^2 - \mu^2}}. \tag{16}$$

The integral over x_0 is performed first. From Eqn. (13) we notice that the source term contains exponentials of the form $\exp \pm i(K_x \pm k_s)x_0$ and therefore $\mu = \pm(K_x \pm k_s)$. Hence, Eqn. (16) for $\mu = K_x + k_s$ yields

if we let $k \sin \theta_s^+ = K \sin \theta + k_s$, then $k^2 - (K \sin \theta + k_s)^2 =$ (17)

$$\sqrt{k^2 - k^2 \sin^2 \theta_s^+} = k \cos \theta_s^+ \text{ and (18) may be written as}$$

$$\int_{-\infty}^{\infty} e^{i(K_x + k_s)x_0} H_0^{(1)}(kr) dx_0 = \frac{2}{k \cos \theta_s^+} e^{i[K_x \sin \theta_s^+ + k \cos \theta_s^+ (z - z_0)]} . \quad (18)$$

In general we can write for the integral for $\mu = \pm (K_x \pm k_s)$

$$\int_{-\infty}^{\infty} e^{\pm i(K_x \pm k_s)x_0} H_0^{(1)}(kr) dx_0 = \frac{2}{k \cos \theta_s^{\pm}} e^{i[\pm K_x \sin \theta_s^{\pm} + k \cos \theta_s^{\pm} (z - z_0)]} , \quad (19)$$

where

$$\frac{\omega}{c} \sin \theta_s^{\pm} = K \sin \theta \pm k_s \quad (20)$$

The integral over z_0 is performed next. The source term contributes exponentials of the form $\exp(\pm iK_z z_0)$. Obtaining the companion exponential from Eqn. (19) the integrals under consideration are

$$\int_{-L}^L e^{i[K_z - k \cos \theta_s^{\pm}]z_0} dz_0 = \frac{2L \sin[K_z - k \cos \theta_s^{\pm}]L}{[K_z - k \cos \theta_s^{\pm}]} \quad (21)$$

and

$$\int_{-L}^L e^{-i[K_z + k \cos \theta_s^{\mp}]z_0} dz_0 = \frac{2L \sin[K_z + k \cos \theta_s^{\mp}]L}{[K_z + k \cos \theta_s^{\mp}]} . \quad (22)$$

Combining now the results of Eqns. (19), (21) and (22), Eqn. (12) yields

$$\psi(x, z, \omega) = -\frac{L \Omega^2 \alpha \psi_0}{2 c^2} \cdot$$

$$\cdot \left\{ \delta(\omega - \Omega - \omega_s) \frac{1}{\frac{\omega}{c} \cos \theta_s^+} e^{i \left[\frac{\omega}{c} x \sin \theta_s^+ + \frac{\omega}{c} z \cos \theta_s^+ \right]} \frac{\sin \left[K_z - \frac{\omega}{c} \cos \theta_s^+ \right] L}{\left[K_z - \frac{\omega}{c} \cos \theta_s^+ \right] L} + \right.$$

$$+ \delta(\omega - \Omega + \omega_s) \frac{1}{\frac{\omega}{c} \cos \theta_s^-} e^{i \left[\frac{\omega}{c} x \sin \theta_s^- + \frac{\omega}{c} z \cos \theta_s^- \right]} \frac{\sin \left[K_z - \frac{\omega}{c} \cos \theta_s^- \right] L}{\left[K_z - \frac{\omega}{c} \cos \theta_s^- \right] L} +$$

$$+ \delta(\omega + \Omega + \omega_s) \frac{1}{\frac{\omega}{c} \cos \theta_s^+} e^{i \left[-\frac{\omega}{c} x \sin \theta_s^+ + \frac{\omega}{c} z \cos \theta_s^+ \right]} \frac{\sin \left[K_z + \frac{\omega}{c} \cos \theta_s^+ \right] L}{\left[K_z + \frac{\omega}{c} \cos \theta_s^+ \right] L} +$$

$$+ \delta(\omega + \Omega - \omega_s) \frac{1}{\frac{\omega}{c} \cos \theta_s^-} e^{i \left[-\frac{\omega}{c} x \sin \theta_s^- + \frac{\omega}{c} z \cos \theta_s^- \right]} \frac{\sin \left[K_z + \frac{\omega}{c} \cos \theta_s^- \right] L}{\left[K_z + \frac{\omega}{c} \cos \theta_s^- \right] L} \left. \right\}. \quad (23)$$

RESULTS

Taking now the inverse Fourier transform we have

$$\psi(x, z, t) = \int_{-\infty}^{\infty} \psi(x, z, \omega) e^{-i\omega t} d\omega \quad (24)$$

which yields after we perform the integration, using Eqn. (23)

$$\psi(x, z, t) = \psi_+ e^{i[\hat{K}_+ \cdot \hat{r} - (\Omega + \omega_s)t]} + \psi_- e^{i[\hat{K}_- \cdot \hat{r} - (\Omega - \omega_s)t]} -$$

$$- \psi_+ e^{i[\hat{\lambda}_+ \cdot \hat{r} - (\Omega + \omega_s)t]} - \psi_- e^{i[\hat{\lambda}_- \cdot \hat{r} - (\Omega - \omega_s)t]}, \quad (25)$$

where

$$\psi_{\pm} = -\frac{L \Omega^2 \psi_0 \alpha}{2 c^2} \frac{\sin \left[\frac{\Omega}{c} \cos \theta - \frac{\Omega \pm \omega_s}{c} \cos \theta_s^{\pm} \right] L}{\frac{\Omega \pm \omega_s}{c} \cos \theta_s^+ \left[\frac{\Omega}{c} \cos \theta - \frac{\Omega \pm \omega_s}{c} \cos \theta_s^{\pm} \right] L} \quad (26)$$

$$\hat{K}_{\pm} \cdot \hat{r} = \frac{\Omega \pm \omega_s}{c} x \sin \theta_s^{\pm} + \frac{\Omega \pm \omega_s}{c} z \cos \theta_s^{\pm} \quad (27)$$

and

$$\hat{Y}_{\pm} \cdot \hat{r} = \frac{\Omega \pm \omega_s}{c} x \sin \theta_s^{\pm} - \frac{\Omega \pm \omega_s}{c} z \cos \theta_s^{\pm} \quad (28)$$

Eqn. (25) shows that the scattered acoustic field is made up of two pairs of plane waves traveling in four different directions. The pair given by the first two terms in Eqn. (25) represents the forward scattered waves while that given by the second two terms represents the backscattered waves.

In addition, from Eqn. (20) we obtain the Bragg formulae. With $k = \pm (\Omega \pm \omega_s)$ we have

$$\sin \theta_s^{\pm} = \frac{\Omega \sin \theta}{\Omega \pm \omega_s} \pm \frac{\omega_s}{\Omega \pm \omega_s} \quad (29)$$

for the forward scattered waves, and

$$\sin \theta_s^{\mp} = -\frac{\Omega \sin \theta}{\Omega \pm \omega_s} \pm \frac{\omega_s}{\Omega \pm \omega_s} \quad (30)$$

for the backscattered waves. Eqns. (29) and (30) give the direction of the scattered waves while Eqn. (25) gives their magnitude.

From Eqn. (26) one finds that the amplitude of the scattered acoustic wave, for nearly normal incidence, is proportional to the perturbation in the acoustic path length (ϕL) caused by the temperature fluctuation.

DISCUSSION AND CONCLUSIONS

When an acoustic wave field crosses a thin layer of thickness $2L$ perturbed by the presence of an internal wave field, the acoustic wave will be modulated by the internal wave and scattering and refraction will occur. Simple analysis shows that the frequency spectrum of the scattered wave is resolved into a forward and a backscattered component. Further, each component is comprised of two side frequencies $(\Omega \pm \omega_s)$ where Ω and ω_s are the angular frequencies of sound and internal waves, respectively. For oceanic conditions and acoustic radiation with carrier frequencies in the audible part of the spectrum, Doppler shifts of a few tens of Hertz are expected. Recent experimental results obtained from Project MIMI appear to indicate the existence of a Doppler shift that has the same shift frequency as that that would have been produced by internal waves in the region.

For the case in which both free surface waves and internal waves are present, the resulting spectrum of the forward (or back) scattered component will be comprised of the fundamental acoustic frequency and four additional side frequencies; the outer two resulting from surface gravity waves and the inner two from internal waves.

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